Analysis of Autocorrelation, Heteroskedasticity,

Non-Stationarity and Over-Reliance on Normality

For Time Series Data

**Group No: 361 Submission for Group Work Project No.2**

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1. **INTRODUCTION**

In this working paper we analyze stock data for their autocorrelation, heteroskedasticity, non-stationarity and over-reliance on normality. We code and run tests for detecting each phenomenon and suggested directions for mitigation or treatment of each phenomenon to either overcome its adverse effects or making advantage of each, on the way to estimating future values of returns, volatility etc. of the data we desire.

1. **AUTOCORRELATION**
   1. **Definition and Description**

According to Smith, autocorrelation is the mathematical representation of the degree of similarity between a given time series and a lagged version of itself over successive time intervals.

* + 1. **Causes of autocorrelation**
* Leaving out relevant exogenous variables in the model.
* Catastrophic natural occurrences such as global pandemics like Covid 19 pandemic, strikes, accidents whose effects can be felt over prolonged periods of time.
* Tendencies of data extrapolation that emanate from traditions and other various forms of behavioral and customary patterns established in the past will often affect affairs in the present moment.
* Manipulation and transformation of data in the form of smoothing, sampling and other treatment methods tend to spread ripple effects over several periods.
* Interconnection of units in an economic system will make effects of a shock that hits one unit to be felt by other units of the same system through their linkages.
  1. **Diagnosis of Autocorrelation: Durbin-Watson Test**
     1. **Diagnosis 1: Durbin-Watson Test**

The Durbin-Watson Test is a measure of autocorrelation (also called serial correlation) in residuals from regression analysis. The DW test statistic is calculated using the following equation:

***d ≈ 2 (1 – r)***

***d =*** Durbin-Watson test statistic value

***r =*** Correlation coefficient

**Assumptions**

The residuals are independent and normally distributed with a mean of zero.

The residuals are stationary.

**Hypotheses**

H0: The is no autocorrelation among the residuals.

H1: The residuals are autocorrelated.

The test statistic has a range between 0 and 4 with the following interpretation:

1. There is no autocorrelation if the test statistic has value of 2.
2. The closer the test statistic is to 0, the more evidence of positive serial correlation.
3. A test statistic value of 4 indicates negative serial correlation.
4. Test statistic values between the range of 1.5 and 2.5 are considered normal (That would mean H0 is accepted, otherwise it is rejected in favor of H1. However, values outside of this range could indicate that autocorrelation would be problematic to the time series and regression models, hence calling for decisive ways to deal with the phenomenon.

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Code listing for OLS fit

**Regression Model of AAPL stock price against three regressor stocks**

To demonstrate the Durbin-Watson test, stock prices data was used and AAPL was regressed on three chosen stocks namely TSLA, META and AMD. The regression model summary results were as follows:

OLS Regression Results

==============================================================================

| Dep. Variable: | | AAPL | R-squared: | | 0.790 | |
| --- | --- | --- | --- | --- | --- | --- |
| Model: | | OLS | Adj. R-squared: | | 0.783 | |
| Method: | | Least Squares | F-statistic: | | 123.8 | |
| Date: | | Sun, 18 Sep 2022 | Prob (F-statistic): | | 2.23e-33 | |
| Time: | | 05:41:04 | Log-Likelihood: | | -243.54 | |
| No. Observations: | | 103 | AIC: | | 495.1 | |
| Df Residuals: | | 99 | BIC: | | 505.6 | |
| Df Model: | | 3 |  | |  | |
| Covariance Type: | | nonrobust |  | |  | |
| ==============================================================================  coef std err t P>|t| [0.025 0.975] | | | | | | |
| Intercept | 30.9480 | 7.551 | 4.099 | 0.000 | 15.966 | 45.930 |
| META | 0.1515 | 0.013 | 11.224 | 0.000 | 0.125 | 0.178 |
| TSLA | 0.1820 | 0.019 | 9.632 | 0.000 | 0.144 | 0.219 |
| AMD | 0.1162 | 0.095 | 1.217 | 0.226 | -0.073 | 0.306 |
| ==============================================================================  Omnibus: 10.854 Durbin-Watson: 0.446 | | | | | | |
| Prob(Omnibus): | 0.004 | | Jarque-Bera (JB): | | 4.392 | |
| Skew: | -0.219 | | Prob(JB): | | 0.111 | |
| Kurtosis: | 2.088 | | Cond. No. | | 1.12e+04 | |

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Tiny code for checking DB independently

A Durbin-Watson test statistic calculated from Python is 0.44575. Since this is outside the range of 1.5 and 2.5, it is not normal. This is positive autocorrelation and its closeness to zero is problematic in the time series model.

* + 1. **Diagnosis 2: Ljung-Box test**

The Ljung-Box test checks the existence of autocorrelation in a time series.

**Hypotheses**

H0: The residuals are independently distributed.

H1: The residuals are not independently distributed; that is, they exhibit serial correlation.

A p-value less than 0.05 leads to rejection of H0 meaning the residuals are not independently distributed or autocorrelation exists. Ljung-Box test performed on built in Python dataset called SUNACTIVITY produced results as follows:

Table

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Code listing for LB test

**lb\_stat lb\_pvalue**

**30** 456.726189 8.528403e-78

With lag 30, the test statistic is 456.726189 and the p-value of the test is 8.528403 x 10-78, which is much less than 0.05. We therefore reject the null hypotheses and conclude that the residuals are not independent. That is to say, autocorrelation exists.

* 1. **ACF AND PACF ANALYSES FOR AAPL STOCK PRICES**

Autocorrelation is an instrumental tool in identifying statistically significant relationships among observed values in linear data such as detecting patterns of periodicity, seasonality or other sources of influence through the use of autocorrelation function (ACF) and partial autocorrelation function (PACF) plots. These are also applied in technical analysis of stock price data.

ACFs are used to visualize time series movement and in predictive analysis as they display a few features which represent data metrics.

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Code listing for plotting ACF

**Typical ACF plot**

The diagram below illustrates an ACF Plot of the AAPL stock prices.

Chart

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The region shaded in blue is the 95% confidence interval in which any value here has no significant correlation with the current or most recent stock price value. The top-marked vertical lines are the lags. These lines represent a specific number of previous values the time series is regressed on. On the AAPL autocorrelation function plot above, previous stock prices beyond day 7 have no significant correlation with the current stock price. The y-axis has the correlation scale from -1 to 1 and each lag has its own correlation value.

**Time Series plot of AAPL stock price with ACF and PACF**

The plots below illustrate the use of autocorrelation in forecasting.

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Code listing for plotting ACF and PACF for AAPL

Chart, histogram

Description automatically generated

However, as a forecasting technique, it also has shortcomings when it comes to data that generates massive amounts of noise. The noise data points will result in skewed correlation metrics that can falsify or fail to capture the desired influence of the autocorrelation model. (Fukushima and Willink)

In light of this, it should be noted that the level of autocorrelation should be ascertained before a time series model is adopted for use for fear that the phenomenon can lead to the following damages:

1. Autocorrelation leads to swelled standard errors and variances resulting in inefficient parameter estimation. There is bias in the determination of the model coefficients.
2. The forecasting power of the model is compromised.
   1. **Treatment of autocorrelation in time series modeling**

If autocorrelation problem is deemed serious enough in time series modeling, then the following options are explored to maintain the integrity of the model's influence and forecasting power:

* 1. Addition of lagged endogenous and/or exogenous variables to the model to treat positive serial correlation.
  2. Addition of seasonal dummy variables to the model to address seasonal correlation
  3. For negative serial correlation, checks for over-differenced variables are done. Otherwise with proper differencing order, negative serial correlation is done away with as illustrated below:
     1. **Treatment by differencing**

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Code listing for differencing

Chart, histogram, box and whisker chart

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From the ACF and PACF of logged AAPL stock price data, it is clear that with first order differencing, all the lagged values are now in the 95% confidence interval region where serial correlation of the values is statistically insignificant. Differencing helps in stabilizing the time series mean by removing the changes in the time series level. This affirms the notion that identifying the right order of differencing is a salve to problematic autocorrelation in time series modeling.

1. **NON-STATIONARITY**
   1. **Definition**

According to Iordanova, non-stationarity is a situation whereby data points in datasets have mean, variances and covariances that change with time. In other words, non-stationarity is the opposite of stationarity. For stationarity the mean function is constant and independent of time, and similarly for stationarity the autocovariance is independent of time for each difference in time.

Non-stationarity is usually common in time series dataset. Time series dataset is a dataset whose data points have date or timestamp attached to them. Time series are common financial datasets such as stock price datasets.

The dataset that usually shows non-stationarity shows the following behaviours or a combination of them:

1. Trends

2. Seasonality (cycles)

3. Random walks

4. Combination of all the three above.

Below we look at each of the above and how to deal with them.

* 1. **Demonstration of non-stationarity in meta stock prices**

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Code listing for META stock data

Chart, line chart

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META stock price of six months

* + 1. **Trend**

The trend behaviour of a non-stationary dataset is where the feature of time series dataset grows over time. From the figure above, we can observe that META stock prices generally keep on increasing from January 2021 to June 2021. Therefore, there’s a trend in this dataset.

* + 1. **Seasonality**

Seasonality is same as cycles and can be defined as a change in a time series dataset over different seasons for example monthly changes. From the graph above, we can observe that there is a cyclic upward movement of the stock prices for example, from mid April-May and May- mid May. This shows that there’s seasonality in META’s stock prices.

* + 1. **Random Walk**

Random walk is a situation whereby the next data values in the time series dataset depend on previous values including their shocks thereby creating a random movement. From the figure above we can observe that the proceeding data values seem influenced by the preceeding data values in that if there’s decline in a day, there’s a higher chance the next day is still in a decline.

* + 1. **The combination of the three behviours**

We can see that the figure above has these three behaviours and therefore is a dataset that shows non-stationarity. In the section below we are going to look at the disadvantages of non-stationarity in dataset and how to handle the associated issues during financial data modeling and forecasting.

* 1. **Issues with non-stationary dataset**

According to Iordanova, as a rule, non-stationary datasets are unpredictable and cannot be modeled or forecasted. Iordanova continues to explain that building a model out of the non-stationary dataset may give results that show an existence of relationship between two variable when in actual sense such a relationship is inexistent. Therefore, to obtain consistent and reliable results, a non-stationary dataset needs to be trans- formed into stationary dataset. Below we look at these transformations.

* 1. **Transforming non-stationary dataset into stationary dataset**

There are several methods for transforming a non-stationary dataset into a stationary dataset, for example, according to Patterson and Mills, differencing or cointegration can be used to remove non-stationarity. Below we use the differencing method to remove non-stationarity in META’s stock prices.

* + 1. **Trend and differencing method**

We can use differencing method to remove trends in datasets. Differencing method involves generating the first-difference time series out of the existing non-stationary time series.

The first difference can be written as below:

∇𝑥𝑡 = 𝑥𝑡 − 𝑥𝑡−1

𝑥𝑡= first observation  
𝑥𝑡−1= the subsequent observation after the first observation

∇𝑥𝑡= first difference

One notable issue with differencing method (from the first difference) above is that an observation is lost when a difference is taken.

* + 1. **Implementing differencing method (with moving average) on META stock prices**

Below we now apply the differencing method to META stock prices. We do this by running two Moving Average (MA) models: one with original META stock prices dataset, then the other with first difference of META’s stock prices. We then compare the results. But, first, let’s have some brief concepts about Moving Average.

Moving Average, (MA) concepts According to Fernando, in technical financial analysis, mov- ing average is used to indicate shocks in price dataset. The moving average can be the model of choice if the shock in the price dataset has an initial impact that gradually fades in a finite time frame. A shock lasting only for a total time period of 𝑞 has a moving average MA (𝑞), mathematically written as follow:

𝑊𝑡 = 𝑊𝑡 +𝜃1𝑊𝑡−1 +𝜃2𝑊𝑡−2 +⋯+𝜃𝑞𝑊𝑡−𝑞

𝜃1, 𝜃2, ⋯ , 𝜃𝑞= parameters  
𝑊𝑡= normally distributed white noise

𝑊𝑡 has mean = 0 and variance 𝜎2 = 1.

We coded below for Moving Average [MA (1)] model with original META stock prices:

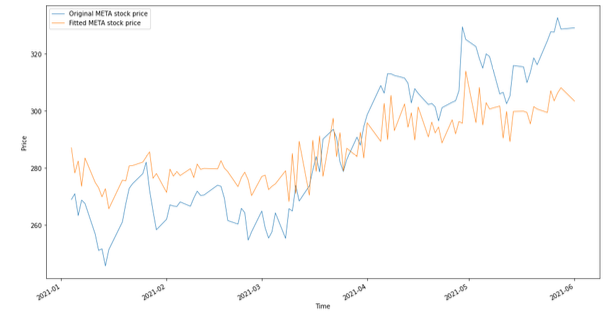
Text

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Code listing Moving Average [MA(1)] model

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Original vs. Moving Average [MA (1)] model fitted META Stock price chart

From the figure above, we can observe that the plot from fitted META stock price values doesn’t closely follow the plot from META stock prices. This is probably because the dataset we used was non-stationary.

Below we run a Moving Average [MA (1)] again but this time with the first difference of META’s stock prices.

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Code listing Moving Average [MA(1)] model with first difference

**Chart, line chart, histogram

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Original vs. Moving Average [MA (1)] model with first difference fitted META Stock price chart

From the above we can see that the curve from the plots of fitted META stock prices closely follow the curve from the plot of original META prices. This shows that transforming our dataset from non-stationarity state to stationarity state using the first differencing improves the model.

1. **OVER-RELIANCE ON NORMALITY**
   1. **Definitions and descriptions**

Before we can discuss the issue of over-reliance normality in financial modeling, it’s important that we define what normality is and what non-normality is, and we do these below.

* + 1. **Normal and Non-normal distributions**

According to Musselwhite and Wesolowski, a normal distribution (also known as a Gaussian distribution) is a hypothetical distribution that’s symmetrical and is used to make comparisons between scores or other types of statistical decisions (2). The normal distribution is a bell-shaped and has mean and median equal. The normal distribution can be constructed using the equation below.

**A picture containing graphical user interface

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A non-normal distribution is any distribution that doesn’t conform to the properties of normal distribution. These commonly include the distributions that show some skewness (as discussed in project 1 under skewness), fat tailed distributions, among others.

* + 1. **Over-reliance on non-normal distributions**

“Over-reliance on normal-distribution” is situation whereby an analyst unnecessarily put emphasis on normal distribution that may negatively affect the results of the model without putting the emphasis on the causes of non-normality and finding alternative models.

Over-reliance on normality can happen at various level, and below we demonstrate the over-reliance on normality by trying to convert the variables by eliminating outliers instead of using methodssuch as robust regressions or weighted least square (WLS) regression. The dataset that we are using is US\_dollar\_index that we used in Project-1 to handle “SENSITIVITY TO OUTLIER” challenge

* + 1. **Demonstration**

First we check for the normality of both exogeneous variable [S&P 500 Index daily return (US\_STK)] and endogenous variable [Dollar Index daily return (DXY)] using QuantileQuantile (QQ) plot and Shapiro Wilk test.

**Graphical user interface, text, application

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Code listing for Shapiro Wilk normality test

**Chart, line chart

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Q-Q plot Shapiro Wilk normality test

**Graphical user interface, text, application

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**Chart, line chart

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From the QQ-plots above, we can see that the two returns are not following normal distribution.

Also from the Shapiro Wilk test the p-values are way below 0.05 so we can reject the null

hypothesis (𝐻0) that the two variables:Dollar Index daily return (DXY) and S&P 500 Index daily

return (US\_STK) follow a normal distribution. From Project-1 under “sensitivity to outliers” we saw that running an ordinary least square (OLS) regression gave results below.

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We know from Project-1 under “Sensitivity to outliers” that the non-gaussian distribution is due to the outliers. However, we could choose to over rely on normality by droping the outliers so that our dataset approximates normal distribution. Below includes the codes for dropping the outliers.

Graphical user interface, text, application, email

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From the test values we can see that the p-value is greater than 0.05 and therefore we accept the null hypothesis (𝐻0 ) that the Dollar Index daily return (DXY) is normally distributed. However, let’s examine the damage.

* 1. **Damage caused by over reliance to non-normality**

We are going to rerun the OLS model, to see observable damages:

Diagram

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Table

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We can observe that the R-squared has dropped from 0.002 to 0.001 when we drop outliers. Similarly, we can also observe that the adjusted R-squared has dropped from -0.002 to -0.004. These two are the observable damages. However, there are also domain-specific damages; for example, removing outliers in financial return leads to wrong modeling since outliers play big part in investment especially if the outliers are purely due to shocks not errors in measurements.

* 1. **Directions**

Instead of dropping the outliers to normalize the variables, the alternative regression modelling that work well with outliers could be used; for example robust regression models such as Mestimation or Weighted Least Square (WLS) regressions can be used. By taking this approach we are focusing on the causes of non-normality than over-relying on normality. In project-1 under “Sentivity to outlier” we demonstrated how to solve this using WLS model, and for the purpose of this paper the model is below.

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**Graphical user interface, text, application, email

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**A screenshot of a computer

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We can observe that using WLS improves the R-squared value from 0.002 to 0.003. This is also three times better than dropping the outliers leave alone the domain specific issues.

1. **HETEROSKEDASTICITY**
   1. **Definition:**

We consider the [linear regression](https://www.wikiwand.com/en/Simple_linear_regression) equation,



where the dependent random variable *yi* equals the deterministic variable *xi* times coefficient *βi* plus a random disturbance term εi  that has mean zero.

Chart, scatter chart

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Graphical interpretation of heteroskedasticity

The disturbances are accepted homoscedastic if the variance of εi is constant σ2; otherwise, they are heteroscedastic. In particular, the disturbances are heteroscedastic if the variance of εi   depends on i or on the value of *xi* . They are said to be heteroscedastic if  σi2 = *xi* σ2 so the variance is proportional to the value of *x*.

* 1. **Description:**

According to White, a sequence of random variables is said to be homoscedastic in statistics if each of its random variables has a fixed variance. Another name for this is homogeneity of variance. Heteroscedasticity is the complementary concept meaning variance can change with respect to the independent variable.

When a variable is heteroscedastic, assuming it to be homoscedastic leads to either unbiased but ineffective point estimates or biased estimates of standard errors or may also cause an overestimation of the goodness of fit as determined by the Pearson coefficient.

As it invalidates statistical tests of significance that presume that the modeling mistakes all have the same variance, the presence of heteroscedasticity is a key concern in regression analysis and the analysis of variance. The generalized least squares estimator should be used instead of the ordinary least squares estimator since it is more effective and remains unbiased in the presence of heteroscedasticity.

For Engle’s research on regression analysis in the presence of heteroscedasticity, which resulted in the creation of the autoregressive conditional heteroscedasticity (ARCH) modeling technique, the econometrician Robert Engle won the 2003 Nobel Memorial Prize for Economics.

* 1. **Demonstration, Diagram and Diagnosis of Heteroskedasticity**

With the python code below, according to Helm et.al, we generate random data with heteroskedasticity:

import numpy as np

import pandas as pd

import matplotlib.pyplot as plt

rng = np.random.RandomState(42)

x = np.linspace(start=0, stop=10, num=1000)

X = x[:, np.newaxis]

y\_true\_mean = 10 + 0.5 \* x

y\_heteroskedastic = y\_true\_mean + rng.normal(loc=0, scale=0.5 + 0.5 \* x, size=x.shape[0])

plt.scatter(X,y\_heteroskedastic, label='heteroskedastic data')

plt.plot(X,y\_true\_mean, '-g', label='true mean')

plt.legend()

plt.show()

Code for creating heteroskedastic data `y\_heteropskedastic`

Chart, scatter chart

Description automatically generated

Plot of heteroskedastic data `y\_heteroskedastic`

There are many tools available to find heteroskedasticity. One of these is the Breusch-Pagan test.

To determine whether heteroscedasticity is present in a regression model, the Breusch-Pagan test is used. The following null and several hypotheses are used in the check:

• There is homoscedasticity, which is the null hypothesis (H0) (the residuals are distributed with equal variance)

• Alternative Hypothesis (HA): There is heteroscedasticity (the residuals are not distributed equally)

We tend to reject the null hypothesis and determine that heteroscedasticity is present in the regression model if the p-value of the test is less than a certain significance level %5. We need to run Breusch-Pagan (BP) test and check the p-value of the test for the null hypothesis of data contains heteroskedasticity. If the p-value is less than 0.05 than heteroskedasticity is detected. Making use of the code presented by Zach, we coded the BP-test as below:

import statsmodels.formula.api as smf

import statsmodels.stats.api as sms

from statsmodels.compat import lzip

df = pd.DataFrame(y\_heteroskedastic)

df2 = pd.DataFrame(X)

df = pd.concat([df,df2], axis=1)

model = smf.ols('y\_heteroskedastic ~ X', data=df).fit()

names = ['Lagrange multiplier statistic', 'p-value', 'f-value', 'f p-value']

test = sms.het\_breuschpagan(model.resid, model.model.exog)

lzip(names, test)

Code for BP Test

output is:

[('Lagrange multiplier statistic', 10.673778282992686),

('p-value', 0.0010866493368004045),

('f-value', 11.710226310110727),

('f p-value', 0.0009081751078028031)]

Result of BP test

Since p-value is 0.001, we can see that heteroskedasticity is present in data “*y\_heteroskedastic*”.

* 1. **Damage Caused by Heteroskedasticity**

White asserts that one of the fundamental tenets of the traditional statistical regression model is the absence of heteroscedasticity. The Gauss-Markov theorem is broken when this assumption is broken, which indicates that OLS estimators are not the best estimators and that their variance is not the lowest of all possible unbiased estimators. The OLS implementation code for our heteroskedastic data is provided below. Let's look at how varying variation causes damage:

import statsmodels.formula.api as smf

import statsmodels.stats.api as sms

from statsmodels.compat import lzip

df = pd.DataFrame(y\_heteroskedastic)

df2 = pd.DataFrame(X)

df = pd.concat( [df,df2], axis=1)

df.columns = ['Y-HET','X']

model = smf.ols('y\_heteroskedastic ~ X', data=df).fit()

model.summary()

Code for OLS Implementation

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Results for OLS

Note that f-statistic is low, R2 value is very low and Jacque Bera result reveals that data was overly non-normal, so we showed that OLS assumptions and Gauss-Markov Theorem cannot be applied here.

Heteroscedasticity doesn't cause normal statistical procedure constant estimates to be biased, though it will cause normal statistical procedure estimates of the variance of the coefficients to be biased, probably on top of or below verity of population variance.

Hypothesis test results are likely to be incorrect due to biased commonplace errors that lead to biased logical reasoning. A researcher can fail to reject a null hypothesis at a certain significance level if, for example, OLS is done on a heteroscedastic information set, providing biased average error estimation, even when that null hypothesis was typical of the given population (making a type II error).

Once correctly standardized and targeted (even when the data doesn't come back from a standard distribution), the OLS computer, under bound assumptions, contains a conventional straight-line distribution. When doing a hypothesis test, this finding is typically used to support using either a Gaussian distribution or a chi square distribution (depending on how the check datum is calculated). Even underneath heteroscedasticity, this exists. With a variance-covariance matrix that is different from the situation of homoscedasticity, the OLS computer in the presence of heteroscedasticity is asymptotically traditional once appropriately normalized and targeted.

The maximum likelihood estimates (MLE) of the parameters will be skewed and inconsistent unless the constructed model properly takes the precise form of heteroscedasticity into account, according to Giles.

* 1. **Directions for overcoming Heteroskedasticity**

Tofallis claims that using a Weighed Least Squares estimation method, in which OLS is applied on revised or weighted values of X and Y, overcomes heteroskedasticity and related effects. Weights are adjusted across observations, usually betting on the fluctuating error variances. Thus, the weights are now directly proportional to the size of the independent variable. Let's use the following code to implement WLS:

# WLS regression result

import statsmodels.api as sm

# Add Absolute residuals and fitted values to dataset columns

df["abs\_residuals"] = np.abs(model.resid)

df["fitted\_values"] = model.fittedvalues

# Fit OLS model with absolute residuals and fitted values

model\_temp = smf.ols("abs\_residuals ~ fitted\_values", data=df).fit()

# Compute weights and add it to the data\_set column

weights = model\_temp.fittedvalues

weights = weights \*\* -2

df["weights"] = weights

# Fit WLS model

X\_N = df['X'].tolist()

Y\_N = df['Y-HET'].tolist()

X = sm.add\_constant(X\_N) # add a intercept point

print(df)

model\_WLS = sm.WLS(Y\_N, X, df["weights"]).fit()

model\_WLS.summary()

Code for WLS Implementation

Table

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Results for WLS (second iteration)

As a result, Weighed Least Squares method came effective in overcoming the heteroskedasticity of data. R2 value improved around %50 and normality of the data is improved as Jacque-Bera test results significantly reduced. However, after the WLS transformation and regression operation, Jacque-Bera result reduced significantly and data became much nearer to a normal distributed one and behaved better for estimation.

1. **CONCLUSION**

In this working paper we analyzed stock data for their autocorrelation, heteroskedasticity, non-stationarity. We had run tests the Augmented Dickey-Fuller, Durbin-Watson and Breusch-Pagan tests for detecting each one. We suggested remedies for each of the phenomena to overcome the adverse effects, as we aim to estimate future values of returns, volatility of the said data.

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